ENGINEERING METHODS OF ANALYSIS OF DEVICES WITH A VIBRATIONALLY FLUIDIZED BED

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A linear rheological model is worked out for the forced oscillations of a granulated bed in a vibration facility, including the working chamber, the associated gas chambers, and the gas distributing grating. Results of the analysis are compared with experimental data.

For the design of production facilities with a granulated bed which work by creating forced oscillations in the charge and for their metrological assurance, knowledge of the dynamic and frequency characteristics of the vibrating charge is necessary. However, purely qualitative concepts about the influence on the said characteristics of the structural elements [1] is insufficient in order to rationally manage hydrodynamical or heat transfer processes in the fluidized bed.

Developing basic assumptions of the theory of linear viscoelasticity [2, 3] as applied to a heterogeneous medium, we will describe the action of the combined elements in a device with a vibrationally fluidized bed, including the working chamber, acoustically closed upper layer, the below-grating volumes of arbitrary size, and the gas—distribution grating (Fig. 1a), with the help of the elementary rheological model presented in the same figure. Rheological models of this type are used in the mechanics of heterogeneous media for the analysis of relaxation phenomena (soil consolidation [4]) and of reciprocating processes connected with the development of autooscillations in pseudofluidized [5] beds and of forced oscillations in vibrationally fluidized beds [6, 7]. The availability of a theory [3] and of an experiment modelling simple systems [8] in combination with the choice of the coefficients according to the results of solutions of the boundary value problems allows one to analytically determine the corresponding dynamical coefficients, except for the impedance coefficient of the grating, which is determined empirically from the linear relation $\Delta P_p = \mu_p \overset{\circledcirc}{X}$. The impedance of the model elements is found from the simple ratios: $E_c = \frac{\pi^2}{4}\frac{P_0}{\varepsilon H_c}$; $E_к = \frac{P_0}{H_к}$; $E_н = \frac{P_0}{H_н}$; the effective viscosity of the layer is $\mu_c = E_c\tau_v$. The impedance of the bed E_c is determined for a quarter-wave distribution of gas pressure pulsation as a function of the height of the bed. Such a distribution is inherent to a layer with nonsymmetric boundary conditions. With symmetry of the dynamic response at the boundaries and approximation of the pressure distribution to a half-wave pattern, the calculational circuit has a different appearance (Fig. 1d, g).

Taking what has been said into account, the forced oscillations of the fluidized bed, with a pressure distribution in the charge tending toward a quarter-wave (Fig. 1a), will be described with an equation whose coefficients include four linear independent relaxation times, one of which τ_v is determined to be connected with interphase friction [9], but the remainder are found to be connected with the resistance of the grating to discharge of the gas surplus from the corresponding elements of the device into the surrounding medium:

$$\tau_v\tau_1\overset{\cdots\cdot}{X} + (\tau_v+\tau_2)\dddot{X} + \left(1+\tau_v\tau_3\frac{\omega_0^2}{\gamma_н}\right)\ddot{X} + (\tau_v+\tau_4)\frac{\omega_0^2}{\gamma_н}\dot{X} + \frac{\omega_0^2}{\gamma_н}X = \ddot{X}_в. \quad (1)$$

According to the data in [10], when the volume of the below-grating chamber is commensurate with the volume of the bed, and when the exhaust speeds are moderate $W \simeq 1$, the impedance of a gas distribution grating with an area of active cross section $S_p > 5\text{-}15\%$ may be thought to be "null," in as much as the hydrodynamic picture practically does not change. It is obvious that for the majority of the devices falling into that class one may neglect the

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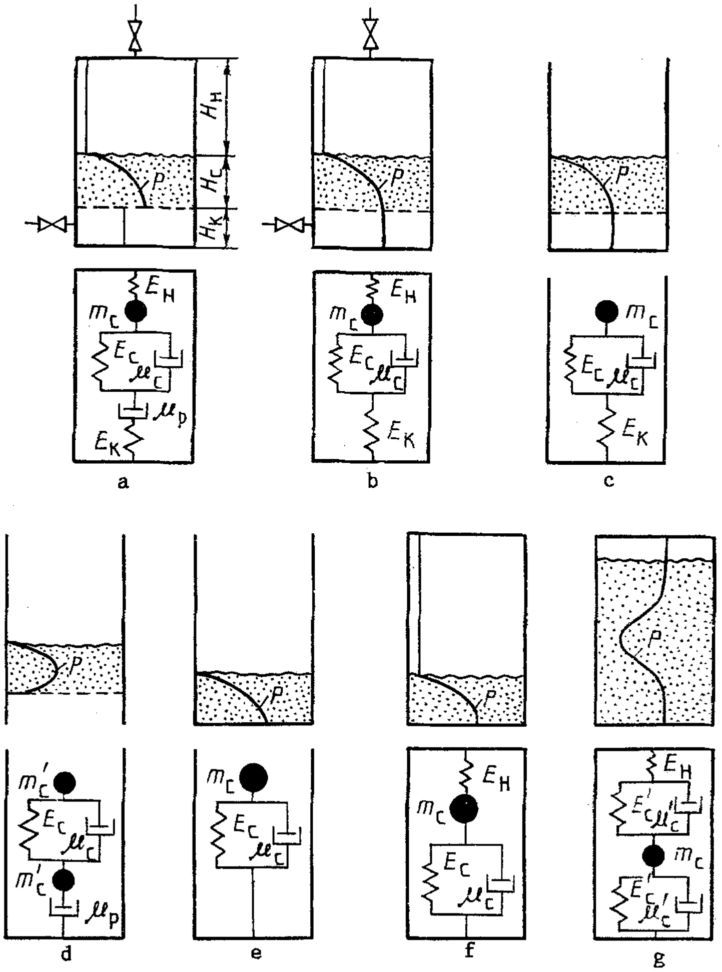

Fig. 1. Schematics of devices with vibratory fluidized beds and the rheodynamic models corresponding to them.

grill impedance, in connection with which the rheological model is transformed (Fig. 1b) and the oscillations of the bed are described with an equation of the second order:

$$\ddot{X} + \frac{\omega_0^2}{1+\gamma_{\text{к}}}\tau_v \dot{X} + \left(\frac{1}{\gamma_{\text{н}}} + \frac{1}{1+\gamma_{\text{к}}}\right)\omega_0^2 X = \ddot{X}_{\text{в}}. \qquad (2)$$

Under the conditions that the perturbing input is described by an harmonic law $X_B = A\cos\omega t$, steady-state oscillations of the bed will correspond to the solution of equation (2):

$$X = \frac{A\Omega^2}{\sqrt{\left(\frac{1}{\gamma_{\text{н}}} + \frac{1}{1+\gamma_{\text{к}}} - \Omega^2\right)^2 + \left(\frac{\theta_v}{1+\gamma_{\text{к}}}\right)^2}}\cos(\omega t - \varphi_1), \qquad (3)$$

where

$$\varphi_1 = \operatorname{arctg}\frac{\frac{\theta_v}{1+\gamma_{\text{к}}}}{\frac{1}{\gamma_{\text{н}}} + \frac{1}{1+\gamma_{\text{к}}} - \Omega^2}. \qquad (4)$$

Having estimated the deformation of the layer, we find the gas pressure drop at the bottom (grating) of the device. In the model concept it will be equivalent to the dynamic tension at the point between corresponding elements of the structural circuit (Fig. 1b) originating during motion of the mass of the bed, and in differential form it is written as

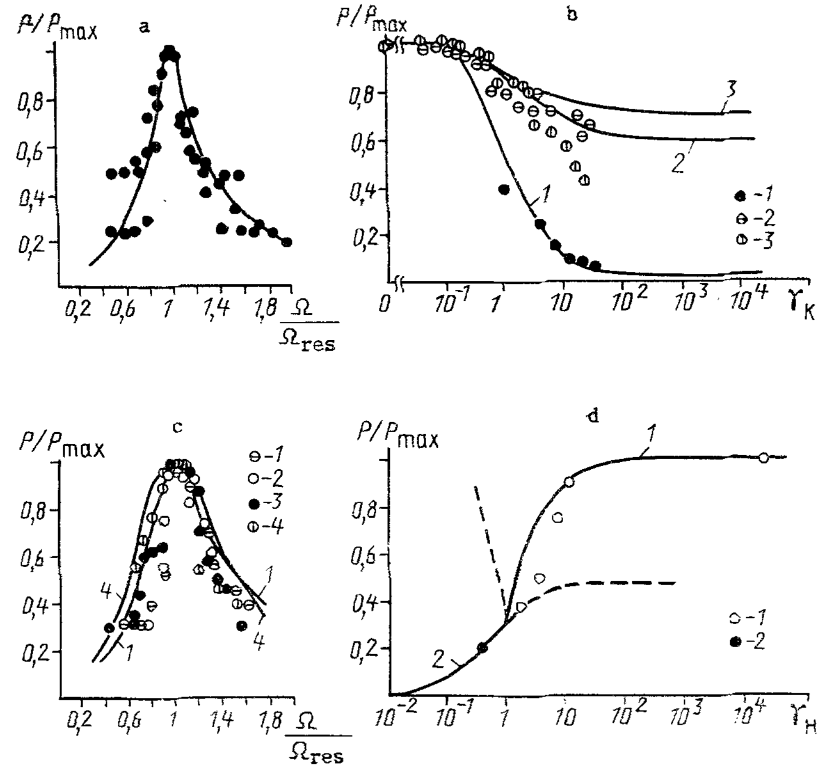

Fig. 2. Dependence of the relative range of the gas pressure pulsation in a vibrationally fluidized bed P/P_{max} upon the assumed frequency of vibration Ω/Ω_{res} (a, c) and upon the relative volume of the additional gas chambers γ (b, d). In Fig. 2a, $\gamma_H = \infty$; $\gamma_K = 8$; $A\omega = 0.37$ to 0.78 m/sec [10, 11]. The curve is calculated according to (7). In Fig. 2b, $\gamma_H = \infty$. The data marked 1 are for $\Omega = 1$, while for 2 and $\Omega = \Omega_{res}$. In 1 and 3 $H_c = 0.07$ m and in 2 it is 0.05 [10, 12]. The curves are calculated according to (7). In 2c, $\gamma_K = 0$, and in curve 1 $\gamma_H = 11.5$, in 2 it is 8.5, and in 3 it is 2.3, and in 4 it is 1.2 [10]. Curve 1 is calculated according to (7), curve 4 is calculated according to (16). In 2d, points 1 have $H_c = 0.12$ M [10], the curve is calculated according to (7). Points 2 have $H_c = 0.24$ m [11], the curve is calculated according (16). $\gamma_K + 0$, $\Omega = \Omega_{res}$.

$$P = \rho_c H_c \left(\frac{\omega_0^2}{1+\gamma_K} \tau_v \dot{X} + \left(\frac{1}{1+\gamma_K} - \frac{1}{\gamma_H} \right) \omega_0^2 X \right). \tag{5}$$

After substitution of the quantity X according to (3) and its derivative into (5) and several reductions we obtain

$$P = P_B \eta_P \cos(\omega t + \varphi_2), \tag{6}$$

where $P_B = \rho_c a_0 A\omega$ is a quantity numerically equal to the pressure which would be developed in the bed if the entire kinetic energy of the charge, moving with a speed $A\omega$, were converted into the potential energy of the compressed gas; η_P is the coefficient of the resonance amplification or dimensionless pressure:

$$\eta_P = \frac{\pi}{2} \frac{\Omega}{1+\gamma_K} \sqrt{\frac{\left(\frac{\gamma_H - 1 - \gamma_K}{\gamma_H} \right)^2 \theta_v^2}{\left(\frac{1}{\gamma_H} + \frac{1}{1+\gamma_K} - \Omega^2 \right)^2 + \left(\frac{\theta_v}{1+\gamma_K} \right)^2}}, \tag{7}$$

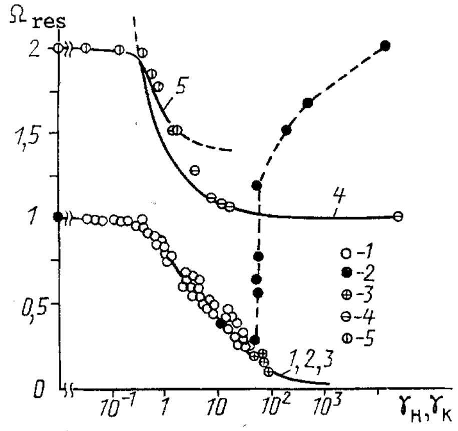

Fig. 3. Dependence of the dimensionless resonant frequency of vibration Ω_{res} corresponding to the maximum range of the gas pressure pulsation upon the relative volume of additional gas cavities γ. In curves 1 to 3 $\gamma_H = \infty$. In 1 H_c = 0.04 to 0.12 m [10]. In 2 it is 0.14 [11], and in 3 it is data from [13]. The curve is calculated according to (7). In 4 and 5 $\gamma_K = 0$. In 4 H_c is 0.07 to 0.27 m [10], the curve is calculated according to (7). In 5 it is 0.23 m [11] and the curve is calculated according to (14).

$$\varphi_2 = \operatorname{arctg} \frac{\theta_v\left(\Omega^2 - \frac{2}{\gamma_H}\right)}{\frac{\gamma_H - 1 - \gamma_K}{\gamma_H}\left(\frac{1}{\gamma_H} + \frac{1}{1+\gamma_K} - \Omega^2\right) - \frac{\theta_v^2}{1+\gamma_K}}. \quad (8)$$

In the same way one may also determine the power dissipated in the bed, which by analogy with the corresponding quantity in electronics we designate active power dissipation:

$$N_a = \int_{T_B} N\,dt, \quad (9)$$

where N is the instantaneous specific power per square meter, transmitted to the layer by the bottom and by the cover:

$$N = N_F + N_K, \quad (10)$$

$$N_F = P\dot{X}_B, \quad (11)$$

$$N_K = P_H\dot{X}_B \quad (12)$$

(P_H is the pressure, developed in the above-layer space). Substituting the values (10)-(12) into (9) and reducing it, we obtain the quantity of power dissipated in the bed:

$$N_a = N_B\eta_N, \quad (13)$$

where $N_B = 1/2\rho_c a_0 A^2\omega^2$ has the dimensions of power, the physical sense of which is determined analogously to P_B: η_N is the dimensionless power:

$$\eta_N = \frac{\pi}{2}\frac{\Omega\theta_v\left(\Omega^2 - \frac{1}{\gamma_H}\right)}{(1+\gamma_K)\left[\left(\frac{1}{\gamma_H}\frac{1}{1+\gamma_K} - \Omega^2\right)^2 + \left(\frac{\theta_v}{1+\gamma_K}\right)^2\right]}. \quad (14)$$

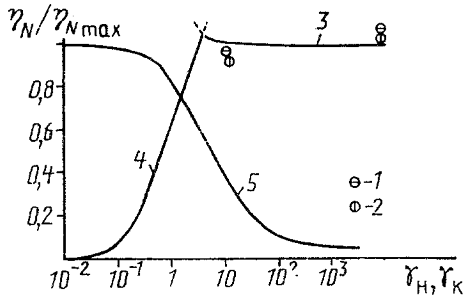

Fig. 4. Dependence of the relative power dissipation in the bed $\eta_N/\eta_{N\max}$ upon the relative volume of the additional gas cavities or chambers γ. In curves 1 to 4 $\gamma_K = 0$ and $H_c = 0.05$ m. In curve 1 $A\omega = 0.16$ m/sec, in 2 it is 0.22, in 3 it is calculated according to (14), in 4 it is calculated according to (17), and in 5 $\gamma_H = \infty$, calculated according to (14).

The solutions of the Eqs. (7) and (14) include a range of particular conditions and are presented in Figs. 2-4 in comparison with experimental data [10-14].

1. $\gamma_H \to \infty$. We consider an open device with a below-grating chamber (Fig. 1c). For moderate volumes of the below-grating chamber, values of pulsating gas pressure at the grating (Fig. 2a, b) and their corresponding resonance frequencies (Fig. 3, curves 1-3) reasonably correspond with the dependences calculated according to (7). A discrepancy is observed for large volumes of the below-grating chamber ($\gamma_K > 10$). Here it is incorrect to neglect the end impedance of the grating. On the other hand, the distribution of pressure with $\gamma_K \sim 100$ is returned to a half-wave (Fig. 1d), and the resonant frequency is successfully in agreement with experiment (Fig. 3). For a pseudofluidized bed located in the regime of autooscillation, an increase of the volume of the below-grating chamber does not change the distribution of pressure in the bed which, as is known, is close to linear, and experimental values of the resonance parameters [13] are coincident with the calculated values.

2. $\gamma_H \to \infty$; $\gamma_K \infty 0$ (Fig. 1e). Equation (2) transforms into an equation of forced oscillations of a bed vibrating on a gas—impermeable floor [8]. As can be seen from Figs. 2b and 3, the additional volumes* (for example, it may be the pneumatic reservoir or the chamber transducer) do not lead to a variation of the resonance parameters of the bed or layer up until $\gamma_K \simeq 0.5$. Additional chamber transducers with a total volume exceeding that value lead to systematic errors of measurement of the pulsating gas pressure, which is difficult to estimate by (7).

3. $\gamma_K = 0$; $\gamma_H \neq \infty$ (Fig. 1f). The device has an impermeable bottom or a grating of large resistance and a restricted upper layer space. The solutions of equations (7) and (14) are applicable in the regions $\gamma_H \geq 10$ (curve 4 in Fig. 3 and curve 1 in Fig. 2d). For small volumes of upper region space ($\gamma_H \to 0$) the value of Ω will grow without limit, which does not correspond with reality and is caused by a change of the pressure distribution to a half-wave. Taking the latter into account, the rheological model changes (Fig. 1g). Here $E_c' = 2E_c$, $\mu_c' = 2\mu_c$, and in connection with it the equation of deformation of the bed for the given model is written as

$$\ddot{X} + 4\omega_0^2\tau_v\dot{X} + 4\frac{1+\gamma_H}{1+2\gamma_H}\omega_0^2 X = \ddot{X}_B. \tag{15}$$

To it correspond the solutions

$$\eta_P = \frac{2\pi\gamma_H\Omega}{(2\gamma_H+1)\sqrt{\left(4\frac{1+\gamma_H}{1+2\gamma_H} - \Omega^2\right)^2 16\theta_v^2}} \tag{16}$$

*Addition of gas chambers other than those in Fig. 1 does not change the basic question, and their influence on the dynamics of the layer may likewise be estimated by Eq. (7).

and

$$\eta_N=\frac{16\pi\gamma_H^2\Omega\theta_v}{(2\gamma_H+1)\left[\left(4\frac{1+\gamma_H}{1+2\gamma_H}-\Omega^2\right)^2+16\theta_v^2\right]}. \tag{17}$$

The plots according to (16) of the dependences are coincident with the experimental data in the regions $\gamma_H \lesssim 2$ (curve 2 Fig. 2d and curve 5 Fig. 3).

Because according to the fragmentary data [14] one cannot speak about a full adequacy of the dependencies plotted according to (14) and (17) with an actual value of η_N, Fig. 4 gives merely a qualitative representation, and along those lines one may judge that the largest amount of energy is transmitted by vibration to the charge when the above-bed space is comparable with the volume of the bed ($\gamma_H \gtrsim 10$) for moderate volumes of additional chambers $\gamma_K \ll 1$, which is necessary to take into account for construction of devices of a similar type.

By means of the given method one may consider also other cases of forced oscillations, for example, when the volume of the below-grating chamber is unboundedly large (see Fig. 1d), which however will be described by a considerably more complicated two-dimensional model. The latter decreases the value of the method and limits its application to single-mass variants.

NOTATION

A, vibration amplitude; a_0, equilibrium speed of sound in the dispersive medium; E, impedance coefficient; H, height; m, mass; N, power; P, gas pressure drop at the floor of the device; P_0, equilibrium pressure of the surrounding medium; S_p, active cross section of the grating; T_B, period of the oscillation; t, time; W, amount of pseudo—liquifaction; X, average deformation of the bed; γ, relative additional volume; ε, porosity; η, coefficient of amplification of the oscillations; $\theta_V = \omega\tau_V$, relative frequency of the forced oscillations of the bed; μ, dynamic viscosity; ρ, density; τ, τ_V, relaxation time and the time of velocity relaxation of the phases [8]; ω, vibration frequency; ω_0, intrinsic oscillation frequency of the bed; and $\Omega = \omega/\omega_0$, dimensionless vibration frequency. The subscripts have the following meanings: a) active; B) vibrational; F) floor; K) chamber; H) upper surface; p) grating or grill; P) pressure; c) bed; res) resonance. Compound quantities are:

$$a_0=\sqrt{\frac{P_0}{\rho_c\varepsilon}};\quad \omega_0=\frac{\pi}{2}\frac{a_0}{H_c};\quad \gamma=\frac{\pi^2}{4}\frac{H}{H_c\varepsilon};$$

$$\tau_1=\mu_p H_K/P_0;\ \tau_2=\frac{\mu_p}{P_0}\left(\frac{4H_c\varepsilon}{\pi^2}+H_K\right);\ \tau_3=\frac{\mu_p}{P_0}\left(\frac{4H_c\varepsilon}{\pi^2}+H_H\right);\ \tau_4=\frac{\mu_p}{P_0}\left(\frac{4H_c\varepsilon}{\pi^2}+H_H+H_K\right).$$

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MODEL OF COAL COMBUSTION IN A FLUIDIZED BED AND ITS EXPERIMENTAL IDENTIFICATION

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The author has formulated a system of one-dimensional steady-state differential equations for the balance of oxidizer, fuel and energy in the diffusion approximation. The model of coal combustion in a fluidized bed is identified from the experimental data, and the unknown parameters of the model describing the rate of oxidation of fuel and the intensity of gas and fuel transfer in the bed are determined.

Fluidization technology opens up possibility of efficient use of a wide range of solid fuels under conditions of increasing ecological requirements to protect the atmosphere. An obstacle to its further development is lack of understanding of the laws of coal combustion in a fluidized bed. The existing combustion models are based mainly on two-phase description of the hydrodynamics of the fluidized system [1-3]. In recent years attempts have been made to determine how large the factors may be, which has led to the development of very cumbersome models [1, 2] that are complex to analyze and especially to relate with the available experimental information on the system examined. It therefore seems desirable to use simpler and physically based models. These contain several unknown parameters which are then determined by comparing the results of theory and experiment (identification of the model) [4]. A simplified model of this kind was proposed in our work [5], where we formulated the equations of oxidizer balance in the continuum and discrete phases of a fluidized bed and the balance equation for the energy of the burning fuel particles. This incomplete model (it lacks the fuel balance equation) was matched or identified using the experimental values of carbon dioxide content in the fuel gases. With it one can analyze a number of regime parameters of the system, assuming that the concentration is constant over the height of the fluidized bed. With a generalized combustion model, based on the full system of fuel balance equations, one can study the fluidized system in greater detail.

We write the full system of balance equations:

for the fuel

$$C'' - m\xi \frac{B}{k} \varphi \text{У} C = 0, \tag{1}$$

for the oxidizer in the continuum phase

$$k\varepsilon \text{У}'' - \frac{\gamma}{N-1} \text{У}' - \xi\varphi BC\text{У} + P\text{П}(\text{У}_{\text{п}} - \text{У}) = 0, \tag{2}$$

for the oxidizer in the discrete phase

$$\frac{N-\gamma}{N-1} \text{У}'_{\text{п}} + P\text{П}(\text{У}_{\text{п}} - \text{У}) = 0 \tag{3}$$

and the energy equation for the hot fuel particles in the bed

$$(C\theta)'' + q\varphi m\xi B\text{У}C - \Phi B \frac{\varphi}{k} \theta C = 0. \tag{4}$$

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